1. D

   $\log_{2018}\sin x + \log_{2018}\cos x + \log_{2018}\tan x = \log_{2018}(\sin x * \cos x * \tan x)$
   Plug in $x = \pi$ and get $\log_{2018}(0 * -1 * 0)$ which is undefined.

2. B

   $((2^3 + 7^0)^{\frac{3}{2}} + (27^{\frac{1}{3}} - 36^{\frac{1}{2}})^2)^{\frac{1}{2}} = ((8+1)^{\frac{3}{2}} + (3-6)^2)^{\frac{1}{2}} = (27+9)^{\frac{1}{2}} = 6$

3. A

   Squaring both sides, we have $(\log_a b)^2 + (\log_b a)^2 + 2 = 16$. So, $(\log_a b)^2 + (\log_b a)^2 = 14$

4. B

   $\frac{1}{\log_7 2} \div \log_{\frac{1}{3}}\frac{1}{9} + \log_8 7 = \log_2 7 \div 2 + \frac{1}{3}\log_2 7 = \frac{5}{6}\log_2 7$

5. E

   Take the base x logarithm of both sides of $x^{\log_{16} x} = 8$ to get $\log_{16} x = \log_x 8$. We can further simplify to $\frac{1}{4}\log_2 x = 3\log_x 2 = \frac{3}{\log_2 x}$, so $(\log_2 x)^2 = \frac{4}{3}$. $\log_2 x = \frac{2}{\sqrt{3}}\ or -\frac{2}{\sqrt{3}}$.
   The product of the solutions would thus be $2^{\frac{2}{\sqrt{3}}} * 2^{-\frac{2}{\sqrt{3}}} = 1$

6. C

   $(\log_x 25)(\log_4 49)(\log_{27} x)(\log_{125} 64)(\log_x 81) = 2$
   $(\log_x x)(\log_4 64)(\log_{27} 81)(\log_{125} 25)(\log_x 49) = 1 * 3 * \frac{4}{3} * \frac{2}{3} * \log_x 49 = 2$
   $\log_x 49 = 2\log_x 7 = \frac{3}{4}$. $\log_x 7 = \frac{3}{8}$, so $\log_7 x = \frac{8}{3}$.

7. B

   $\sqrt{2 + \sqrt{2^2 + \sqrt{2^4 + \sqrt{2^8 + \cdots}}}}$ simplifies to $\sqrt{2(1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}$. Now, solving for the inside part $x = 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$, we get $x - 1 = \sqrt{x}$. The quadratic formula gives $x = \frac{3 \pm \sqrt{5}}{2}$ but the negative sign gives us an x value less than 1 where x is clearly greater than 1 so we have $x = \frac{3+\sqrt{5}}{2}$. Our final answer is $\sqrt{2\left(\frac{3+\sqrt{5}}{2}\right)} = \frac{\sqrt{10}+\sqrt{2}}{2}$
   $|10 - 2| = |2 - 10| = 8$.

8. A

   $\ln(1 + \log_2(3 + \log_4(5 + x))) = 0$
   $1 + \log_2(3 + \log_4(5 + x)) = 1$
   $3 + \log_4(5 + x)) = 1$
   $\log_4(5 + x)) = -2$
   $x = 4^{-2} - 5 = -\frac{79}{16}$

9. D

   $\log_3 r + \log_3 s + \log_3 t = \log_3 rst$. The product of the roots is $-\frac{-3}{9} = \frac{1}{3}$, so $\log_3 rst = \log_3 \frac{1}{3} = -1$.

10. B

    Note that $-2 = \sqrt[5]{-32}$, so x will be slightly less than -2.

I. False because of the above

II. True because slightly less than -2 cubed less than -2 cubed

III. True because $\sqrt{-(-2)}$ is already greater than 1

IV. False because slightly less than -2 squared is slightly greater than -2 squared

V. True because slightly less than -2 to the fourth will be greater than 2 to the fourth

11. A

$S = \frac{1}{1+r_1^2} + \frac{1}{1+r_2^2} + \frac{1}{1+r_3^2} + \cdots + \frac{1}{1+r_{20}^2}$. We can use difference of squares to create two more manageable sums:

$$S = \frac{\left(\frac{1}{r_1 - i} + \frac{1}{r_2 - i} + \frac{1}{r_3 - i} + \cdots + \frac{1}{r_{20} - i}\right) - \left(\frac{1}{r_1 + i} + \frac{1}{r_2 + i} + \frac{1}{r_3 + i} + \cdots + \frac{1}{r_{20} + i}\right)}{2i}$$

The first sum can be calculated with a new polynomial $(x+i)^{20} - 7(x+i)^3 + 1$. Now we can use binomial theorem to get $\frac{-21+20i}{2+7i} = \frac{98+187i}{53}$.

The second sum can be calculated with a new polynomial $(x-i)^{20} - 7(x-i)^3 + 1$. Now we can use binomial theorem to get $\frac{-21-20i}{2-7i} = \frac{98-187i}{53}$

So we have $S = \frac{\frac{98+187i}{53} - \frac{98-187i}{53}}{2i} = \frac{187}{53}$. 187-53=134

12. D

$(\log_{2a} 4^x)(1 + \log_2 a) = (\log_{2a} 4^x)(\log_2 2a) = (\log_{2a} 2a)(\log_2 4^x) = 2x$

13. C

$3^{x-4} = 4^{x-3}$

$(x-4)\ln 3 = (x-3)\ln 4$

$x \ln 3 - 4\ln 3 = x\ln 4 - 3\ln 4$

$x(\ln 3 - \ln 4) = \ln 81 - \ln 64$

$$x = \frac{\ln 81 - \ln 64}{\ln 3 - \ln 4} = \frac{\ln 64 - \ln 81}{\ln 4 - \ln 3}$$

14. D

First, we must multiply $F(x)$ by 8 to get $2^{x+3} + 56$. Then, we add 8 to get $2^{x+3} + 64$.

15. C

$\frac{\log n}{2\log m} + \frac{\log m}{2\log n} = 1$. Substituting $x = \log_m n$, we have $x + \frac{1}{x} = 2$ with the only solution of $x = 1$. Thus, $n = m$.

16. D

$a\log_{1440} 5 + b\log_{1440} 2 + c\log_{1440} 3 = d$

$\log_{1440} 5^a + \log_{1440} 2^b + \log_{1440} 3^c = d$

$5^a 2^b 3^c = 1440^d = 5^d 2^{5d} 3^{2d}$

Thus, $a = 1, b = 5, c = 2, d = 1.\ 1*5 + 2*1 = 7$

17. E

$$\sqrt{6 + (1 + \sqrt{3 + (1 + \sqrt{3 + \sqrt{8}})^2})^2} = \sqrt{6 + (1 + \sqrt{3 + (1 + (1 + \sqrt{2}))^2})^2}$$

$$\sqrt{6 + \left(1 + \sqrt{3 + \left(2 + \sqrt{2}\right)^2}\right)^2} = \sqrt{6 + \left(1 + \sqrt{9 + 4\sqrt{2}}\right)^2} = \sqrt{6 + \left(2 + 2\sqrt{2}\right)^2} =$$

$4 + \sqrt{2}$. 4+2=6.

18. B

$\log_x y + \frac{1}{\log_x y} = \frac{10}{3}$. We can use substitution a quadratic to solve this or recognize that the reciprocals $\frac{1}{3} + 3 = \frac{10}{3}$. So, $\log_x y = \frac{1}{3}$ and $x = y^3$. Now we have, $y^4 = 400$ and $y = 2\sqrt{5}$. Thus, $x = \frac{400}{2\sqrt{5}} = 40\sqrt{5}$. $40\sqrt{5} - 2\sqrt{5} = 38\sqrt{5}$.

19. D

Simplify $\log_4(\log_{64} x) = \log_{64}(\log_4 x)$ to $\frac{1}{2}\log_2(\frac{1}{6}\log_2 x) = \frac{1}{6}\log_2(\frac{1}{2}\log_2 x)$. Substitute a $= \log_2 x$ and get $\frac{1}{2}\log_2(\frac{1}{6}a) = \frac{1}{6}\log_2(\frac{1}{2}a)$ or $\log_2((\frac{1}{6}a)^3) = \log_2(\frac{1}{2}a)$ or $\frac{1}{216}a^3 = \frac{1}{2}a$. Solving, we get $a^2 = 108$.

20. C

Using approximations of $\log_{10} 2$ and $\log_{10} 3$ where only up to 3 decimal places are needed, take the base 10 logarithm of $18^{50} = 3^{100}2^{50}$ to get $100(0.477) + 50(0.301) = 62.75$. $12^{50} = 10^{62.75}$, thus the number of digits is 62.75 rounded up or 63.

21. A

$(3 + \sqrt{7})^6 + (3 - \sqrt{7})^6 = 2(3^6 + 15 * 3^4 * 7 + 15 * 3^2 * 7^2 + 7^3) = 32384$.

We know that $(3 - \sqrt{7})^6 < 1$, so $32384 - 1 < (3 + \sqrt{7})^6 < 32384$. Thus the greatest integer less than $(3 + \sqrt{7})^6$ would be $32384 - 1 = 32383$. 3+2+3+8+3=19

22. C

Using basic matrix identities, we have $\left(-\frac{e^5}{e^9}\right) + \left(-\frac{e^5}{e^9}\right) = -2e^{-4}$.

23. B

The units digit of 3^n repeats after every fourth number with 3, 9, 7, 1. In mod4, 5^{7^9} is $1^{7^9} = 1$, so the units digit of will be the first number in the cycle, so 3.

24. B

We can ignore the tens, hundreds, and thousands digits of all number in both summations. We have

$(1^{2017} + 2^{2017} + 3^{2017} + \cdots + 7^{2017}) + (1^{2018} + 2^{2018} + 3^{2018} + \cdots + 8^{2018})$

2017 is 1mod4 and 2018 is 2mod4. Thus, we can simplify because the units digits repeat every 4th power.

$(1^1 + 2^1 + 3^1 + \cdots + 7^1) + (1^2 + 2^2 + 3^2 + \cdots + 8^2)$.

The first part can be grouped up by 10 numbers (summing 0 through 9 to get 45 which has a units digit of 5). 2017 divided by 10 is 201 remainder 7 which gives us $201 * 5 + 1 + 2 + 3 + 4 + 5 + 6 + 7 = 1033$, so the first sum has a units digit of 3.

The second part can still be grouped up by 10 numbers with some simplification. We have $0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 285$ which also has a remainder of 5. Thus, $201 * 5 + 0 + 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 1209$, so the second sum has a units digit of 9.

3+9=12, so our answer is 2.

25. A

Take mod 5 of F(n) and get $1 + (-1)^n + 2^n + (-2)^n$ so whenever n is odd, then we get 0. There are 1009 odd integers.

Now, if n is even, we can plug in n=2x to get $1 + 1^x + (-1)^x + (-1)^x$ so only odd values of x work so we have 505 odd values of x from 1 to 1009. 1009+505=1514.

26. C

After 12 minutes, he's left with $6.4kg * \left(\frac{1}{2}\right)^{\frac{720\,seconds}{144\,seconds}} = 0.2kg = 200g$ of Zn-7, which means he has 175g of Zn-71. $2800g * \left(\frac{1}{2}\right)^x = 175g$ gives us $x = 4$. $\frac{720\,seconds}{4} = 180\,seconds.$

27. E

$\cos\left(\pi\sqrt{x^2+7}\right) - 1$ is restricted to be greater than or equal to 0 because of the root. $\cos\left(\pi\sqrt{x^2+7}\right) - 1 \geq 0$, $\cos\left(\pi\sqrt{x^2+7}\right) \geq 1$. The maximum value of a cosine function is 1 so $\cos\left(\pi\sqrt{x^2+7}\right) = 1$ with the restriction that $\sqrt{x^2+7}$ is an even integer. Thus, $\log_2(-x^2+7x-10) = 1$, $x^2 - 7x + 12 = 0$ so $x = 3\ or\ 4$. Only $x = 3$ satisfies the $\sqrt{x^2+7}$ restriction.

28. D

The vertical asymptote is where $1 - 4x = 0$ or $x = \frac{1}{4}$

The x-intercept is where $\text{f(x)} = 2 - \log_3(1 - 4\text{x}) = 0$. $x = -2$, so the shortest distance between (-2,0) and the line $x = \frac{1}{4}$ would be along the x axis or simply $|-2| + \left|\frac{1}{4}\right| = \frac{9}{4}$

29. C

We have a telescoping series with $\sqrt[3]{9 \cdot \sqrt[5]{81 \cdot \sqrt[7]{729 \cdot \sqrt[9]{6561 \ldots}}}} = 9^{\frac{1}{3}} * 9^{\frac{2}{3*5}} * 9^{\frac{3}{3*5*7}} \ldots$ The telescoping series $\frac{1}{3} + \frac{2}{3*5} + \frac{3}{3*5*7} + \frac{4}{3*5*7*9}$ ... can be expressed as $\sum_{n=1}^{\infty} \frac{n}{(2n+1)!!}$ which converges to ½ so 9^½ = 3.

30. A

We essentially such a series:

$S = 1 * 2 + 2 * 2^2 + 3 * 2^3 + \cdots + 100 * 2^{100}$

Now, we can manipulate this series by multiplying by 2 to get

$2S = 1 * 2^2 + 2 * 2^3 + 3 * 2^4 + \cdots + 100 * 2^{101}$

Subtracting, we have

$-S = 2 + 2^2 + 2^3 + \cdots + 2^{100} - 100 * 2^{101}$

Or

$-S = 2^{101} - 100 * 2^{101}$, so $S = 99 * 2^{101}$

Taking the base 2 log of S, we get $101 + \log_2 99$ while the restriction prevents the expressing from being manipulated. 101+2+99=202.